From (2),

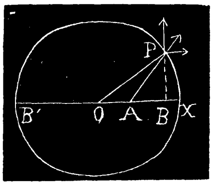
$$\sin\theta=\frac{G\cos\varphi}{g}=\frac{32.05209\times.731}{32}=.73219.$$

$\therefore\ \theta=47^\circ\ 4'\ 15''$.

$\therefore$ Difference in apparent weight$=481.0864\cos(47^\circ\ 4'\ 15'')=327.6583$ lbs.

II Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

This problem depends on the velocities in space of the Express east and the Express west.



In the figure, let P be the point of the train on the 47th parallel, O the center of the earth, AP the normal at P, φ the angle PO makes with the equatorial diameter $B'x$, θ the angle the normal makes with the same line, (θ is the latitude of P).

Let $OB=\rho$, $OP=r$, 300 tons$=W$, f=centrifugal force in the direction AP (vertical direction), g=gravity on 47th parallel, G=gravity at equator. $a=6377377$ meters$=20923536$ feet=equatorial radius, e=ellipticity of the earth.

$\rho=r\cos\varphi=a\cos\theta/\sqrt{(1-e^2\sin^2\theta)}$.

Now $\theta=47^\circ$, $e^2=.006920928$. $\therefore\ \rho=4357445.45$ meters.

One day=86400 seconds.

$\therefore\ 2\pi\rho/86400=316.8831$ meters$=1039.37$ feet per second, the velocity of P due to the earth's rotation.

60 miles an hour=88 feet per second.

$1039.37-88=951.37$, the train's velocity in space going west.

$1039.37+88=1127.37$, the train's velocity in space going east.

$f=Wv^2/g\rho=Wv^2/gr\cos\varphi$.

$\therefore\ F=f\cos\theta=Wv^2\sqrt{(1-e^2\sin^2\theta)}/ag$.

Now $g=G(1+\frac{1}{4}e^2\sin^2\theta)$.* $G=32.2015235$ feet.

$\therefore$, $g=32.23130991$ feet per second. $\therefore\ F=.000000444v^2$ tons.

$\therefore\ F=.4018399$ tons going west. $F=.5643076$ tons going east.

$\therefore$ Difference$=.1624657$ tons$=324.9354$ pounds.

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*This expression for the true value of gravity in latitude θ is new to me. If any reader of the MONTHLY can tell me where to find it used previously, and by whom, I would be greatly pleased to know. I believe it to be new and unused before.

AVERAGE AND PROBABILITY.

76. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

In a given ellipse, the extremities of a focal chord are joined with the center. Find the average area of the angle thus formed.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metalurgy, Rolla, Mo.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; J. SCHEFFER, A. M., Hagerstown, Md.; and L. C. WALKER, Instructor in Mathematics, Leland Stanford, Jr., University, Palo Alto, Cal.

Let θ=the angle the focal chord makes with the major axis.

The length, l, of this focal chord

$$=\frac{a(1-e)^2}{1+e\cos\theta}+\frac{a(1-e^2)}{1+e\cos\theta}=\frac{2a(1-e^2)}{1-e^2\cos^2\theta}.$$

The perpendicular distance from the center to this chord$=ae\sin\theta$.

$\therefore$ Area of triangle=

$$A=\frac{a^2e(1-e^2)\sin\theta}{1-e^2\cos^2\theta}.$$

I. When the chords are drawn at equal angular intervals,

$$\triangle=\frac{\int_0^{\frac{1}{2}\pi}Ad\theta}{\int_0^{\frac{1}{2}\pi}d\theta}=\frac{2a^2e(1-e^2)}{\pi}\int_0^{\frac{1}{2}\pi}\frac{\sin\theta d\theta}{1-e^2\sin^2\theta}=\frac{a^2(1-e^2)}{\pi}\log\left(\frac{1+e}{1-e}\right).$$

DEAN, DRANE, SCHEFFER, WALKER.

II. When the abscissas of the extremity of the chord are drawn at equal intervals,

$$\triangle=\frac{\int Adx}{\int dx}=\frac{\int_0^{\frac{1}{2}\pi}\frac{A\sin\theta d\theta}{(1-e\cos\theta)^2}}{\int_0^{\frac{1}{2}\pi}\frac{\sin\theta d\theta}{(1-e\cos\theta)^2}}=a^2e(1-e)^2(1+e)\int_0^{\frac{1}{2}\pi}\frac{\sin^2\theta d\theta}{(1-e\cos\theta)^3(1+e\cos\theta)}$$

$$=-\tfrac{1}{2}a^2(1-e)^2(1+e)+\tfrac{1}{2}a^2(1-e)^2(1+e)\int_0^{\frac{1}{2}\pi}\frac{(e+\cos\theta)d\theta}{(1-e^2\cos^2\theta)^2}$$

$$=\frac{a^2(1-e)}{8b}\left(\pi a^3e^3-4b(1-e^2)+20a^2\sqrt{1-e^2}+\frac{2a^2}{e}\sin^{-1}e\right).$$

III. When the chord varies with the arc,

$$\triangle=\frac{\int Ads}{\int ds}.$$

ZERR.